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THE PRICING OF POWER QUANTO OPTIONS UNDER STOCHASTIC VOLATILITY

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Abstract. In this paper, we use the stochastic volatility model introduced by Schöbel and Zhu [7] to drive a closed-form expression for the price of a European power quanto call option.

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1. Introduction

Power options are a class of exotic options in which the payoff at maturity is related to the certain positive power of the underlying asset price, which allows investors to provide high leverage strategy and to hedge nonlinear price risks. Pricing power options based on the classical Black-Scholes [2] model, which is relatively easy, has a problem of assuming a constant volatility which leads to smiles and skews in the implied volatility of the underlying asset. For that reason, it is natural to consider a stochastic volatility model in valuing power options. Stochastic volatility models, such as Hull-White [4], Stein-Stein [8], Heston [3] and Schöbel-Zhu [7], are frequently used in pricing various kinds of European options. Despite its importance, very few researches have been done on pricing power options using stochastic volatility models primarily due to the sophisticated stochastic process for underlying assets, the more complex payoff structure and volatilities and the difficulty of finding an analytic form of the option price.

To mention some of the works on pricing power options with stochastic volatilities, Bakshi and Madan [1] come across a squared power payoff in a general diffusion setup. Kim et al. [6] derive semi-analytic solutions for power option prices under the Heston model. Ibrahim et al. [5] derive the PDE from the Heston model for power options and solve the PDE for the characteristic function, and then apply the fast Fourier transform technique to price the power option under the Heston model.

A quanto is a type of financial derivative whose pay-out currency differs from the natural denomination of its underlying financial variable. A quanto option has both the strike price and the underlying asset price denominated in foreign currency. At exercise, the value of the quanto option is calculated as the option's intrinsic value in the foreign currency, which is then converted to the domestic currency at the fixed exchange rate. Since power options become widely used in derivative markets on foreign exchange, the combination of power option and quanto option can be seriously considered on its pricing.

In this paper, we drive a closed-form expression for the price of a European power quanto option under the Schöbel-Zhu [7] stochastic volatility model. The Schöbel-Zhu model allows that the volatility follows an Ornstein-Uhlenbeck process which has a mean reversion property and is correlated with the return on asset. One of the major advantage of using the Schöbel-Zhu model over the Heston model is the accessibility of closed-form expressions for the prices of European power quanto options which appear this paper. We can here apply the power option to the quanto option pricing by using the quanto measure.

In section 2, we first specify the quanto dynamics of the processes of underlying asset and its volatility under the Schöbel-Zhu model. In section 3, using the results of previous section, we get a closed-form expression for the price of a European power quanto option under the Schöbel-Zhu model. Theorem 3.3 is the main result of the paper.

2. A model specification

For a dividend paying asset with the yield rate q, we assume the process of the asset price S_t to be denominated in foreign currency X and to have the following dynamics:

$$dS_t = \left(r^X - q\right) S_t dt + v_t S_t dB_t^{\mathbb{Q}^X}, \tag{1}$$

$$dv_t = \kappa\left(\theta - v_t\right) dt + \xi dW_t^{\mathbb{Q}^X} \tag{2}$$

under the risk-neutral probability measure $\mathbb{Q}^X$, where $B_t^{\mathbb{Q}^X}$ and $W_t^{\mathbb{Q}^X}$ are two correlated $\mathbb{Q}^X$-standard Brownian motions. Also, r^X is the foreign riskless rate and v_t follows the stochastic volatility process of Schöbel-Zhu [7] model for the asset price S_t with constant parameters κ, θ and ξ. Moreover, we assume that the investor whose domestic currency is Y and who wishes to obtain exposure to the asset price S_t without carrying the corresponding foreign exchange risk.

Let $Z_t^{Y/X}$ denote the price of one unit of currency Y in units of currency X and we assume that $Z_t^{Y/X}$ follows the standard Black-Scholes type dynamics under $\mathbb{Q}^X$ such as

$$dZ_t^{Y/X} = \left(r^X - r^Y\right) Z_t^{Y/X} dt + \sigma_{\mathrm{FX}} Z_t^{Y/X} d\hat{B}_t^{\mathbb{Q}^X},$$

where $\hat{B}_t^{\mathbb{Q}^X}$ is a $\mathbb{Q}^X$-standard Brownian motion. Also, r^Y is the domestic riskless rate and σ_{FX} is the constant volatility of the foreign exchange rate $Z_t^{Y/X}$. Furthermore, this model allows three constant correlations ρ, ν and β satisfying

$$dB_t^{\mathbb{Q}^X} dW_t^{\mathbb{Q}^X} = \rho dt, \quad dB_t^{\mathbb{Q}^X} d\hat{B}_t^{\mathbb{Q}^X} = \nu dt, \quad dW_t^{\mathbb{Q}^X} d\hat{B}_t^{\mathbb{Q}^X} = \beta dt.$$

Using the change of measure from $\mathbb{Q}^X$ to the domestic risk-neutral probability measure $\mathbb{Q}^Y$ with the Radon-Nikodým derivative

$$\left.\frac{d\mathbb{Q}^Y}{d\mathbb{Q}^X}\right|_{\mathcal{F}_t} = \frac{Z_t^{Y/X}}{Z_0^{Y/X}} e^{\left(r^Y - r^X\right)t} = e^{-\frac{1}{2}\sigma_{\mathrm{FX}}^2 t + \sigma_{\mathrm{FX}} \hat{B}_t^{\mathbb{Q}^X}},$$

the Girsanov's theorem implies that the processes $B_t^{\mathbb{Q}^Y}$, $W_t^{\mathbb{Q}^Y}$ and $\hat{B}_t^{\mathbb{Q}^Y}$ defined by

$$\begin{aligned} dB_t^{\mathbb{Q}^Y} &= dB_t^{\mathbb{Q}^X} - \nu\sigma_{\text{FX}}dt, \\ dW_t^{\mathbb{Q}^Y} &= dW_t^{\mathbb{Q}^X} - \beta\sigma_{\text{FX}}dt, \\ d\hat{B}_t^{\mathbb{Q}^Y} &= d\hat{B}_t^{\mathbb{Q}^X} - \sigma_{\text{FX}}dt \end{aligned}$$

are again $\mathbb{Q}^Y$-standard Brownian motions, so called the *quanto measure*. Thus, the foreign exchange rate $Z_t^{X/Y}$ denoting the price in currency X per unit of the domestic currency Y follows the stochastic process:

$$dZ_t^{X/Y} = \left(r^Y - r^X\right) Z_t^{X/Y} dt - \sigma_{\text{FX}} Z_t^{X/Y} d\hat{B}_t^{\mathbb{Q}^Y}.$$

Also, we obtain the following dynamics of S_t and v_t under $\mathbb{Q}^Y$:

$$dS_t = \left(r^X - q + \nu\sigma_{\text{FX}}v_t\right) S_t dt + v_t S_t dB_t^{\mathbb{Q}^Y}, \tag{3}$$

$$dv_t = \kappa\left(\hat{\theta} - v_t\right) dt + \xi dW_t^{\mathbb{Q}^Y} \tag{4}$$

with $\hat{\theta} = \theta + \frac{\beta\sigma_{\text{FX}}\xi}{\kappa}$. We notice that (4) maintains the same form as (2).

3. A closed-form expression

Here, using the model specified in previous section, we drive a closed-form expression for the price of a European power quanto option. Before we investigate the pricing of power quanto options, we first compute the value of a power quanto forward contact $\mathbb{E}_{\mathbb{Q}^Y}\left[S_T^\alpha \middle| \mathcal{F}_t\right]$ for $\alpha > 0$ from the risk-neutral method. The following two lemmas are about a conditional expectation under the measure $\mathbb{Q}^Y$, both of which are crucial ingredients to the main result of the paper.

Lemma 3.1. *Under the assumptions of (3) and (4) with $\alpha > 0$, we get the following equality:*

$$\begin{aligned} \mathbb{E}_{\mathbb{Q}^Y}\left[S_T^\alpha \middle| \mathcal{F}_t\right] = S_t^\alpha e^{\alpha\left(r^X - q\right)(T-t) - \frac{\rho\alpha}{2\xi}\left\{v_t^2 + \xi^2(T-t)\right\}} \\ \times \mathbb{E}_{\mathbb{Q}^Y}\left[e^{-\int_t^T\left(c_1 v_s^2 + c_2 v_s\right)ds + c_3 v_T^2} \middle| \mathcal{F}_t\right], \end{aligned}$$

where c_1, c_2 and c_3 are constants with

$$c_1 = \alpha\left\{\frac{1 - \alpha\left(1-\rho^2\right)}{2} - \frac{\rho\kappa}{\xi}\right\}, \quad c_2 = \alpha\left(\frac{\rho\kappa\hat{\theta}}{\xi} - \nu\sigma_{FX}\right), \quad c_3 = \frac{\rho\alpha}{2\xi}.$$

Proof. From (3), we get

(5)

$$\mathbb{E}_{\mathbb{Q}^Y}\left[S_T^\alpha \middle| \mathcal{F}_t\right] = S_t^\alpha e^{\alpha\left(r^X - q\right)(T-t)}$$

$$\times \mathbb{E}_{\mathbb{Q}^Y}\left[e^{\frac{\alpha}{2}\left\{\alpha\left(1-\rho^2\right)-1\right\}\int_t^T v_s^2 ds + \alpha\nu\sigma_{\text{FX}}\int_t^T v_s ds + \rho\alpha\int_t^T v_s dW_s^{\mathbb{Q}^Y}} \middle| \mathcal{F}_t\right] \tag{6}$$

by writing $B_t^{\mathbb{Q}^Y} = \rho W_t^{\mathbb{Q}^Y} + \sqrt{1-\rho^2} W_t$ with W_t as a $\mathbb{Q}^Y$-standard Brownian motion independent of $W_t^{\mathbb{Q}^Y}$ and using the tower property. Applying the

Itô formula to v_t^2, we have

$$dv_t^2 = 2\kappa\left(\frac{\xi^2}{2\kappa} + \hat{\theta}v_t - v_t^2\right)dt + 2\xi v_t dW_t^{\mathbb{Q}^Y},$$

which implies that

(7)
$$\int_t^T v_s dW_s^{\mathbb{Q}^Y} = \frac{1}{2\xi}\left\{v_T^2 - v_t^2 - \xi^2(T-t) - 2\kappa\hat{\theta}\int_t^T v_s ds + 2\kappa\int_t^T v_s^2 ds\right\}.$$

Substituting (7) into (5), we obtain

$$\mathbb{E}_{\mathbb{Q}^Y}\left[S_T^\alpha\middle|\mathcal{F}_t\right] = S_t^\alpha e^{\alpha\left(r^X-q\right)(T-t)-\frac{\rho\alpha}{2\xi}\left\{v_t^2+\xi^2(T-t)\right\}}\mathbb{E}_{\mathbb{Q}^Y}\left[e^{-\int_t^T\left(c_1v_s^2+c_2v_s\right)ds+c_3v_T^2}\middle|\mathcal{F}_t\right]$$

with

$$c_1 = \alpha\left\{\frac{1-\alpha\left(1-\rho^2\right)}{2} - \frac{\rho\kappa}{\xi}\right\},\quad c_2 = \alpha\left(\frac{\rho\kappa\hat{\theta}}{\xi} - \nu\sigma_{\mathrm{FX}}\right),\quad c_3 = \frac{\rho\alpha}{2\xi}.$$

□

Now, we need the following result of Schöbel and Zhu [7] to get the detailed value of $\mathbb{E}_{\mathbb{Q}}\left[S_T^\alpha\middle|\mathcal{F}_t\right]$ mentioned above.

Lemma 3.2. *Under the assumption of (2), together with constants c_1, c_2 and c_3, we get the following equality:*

$$\mathbb{E}_{\mathbb{Q}}\left[e^{-\int_t^T\left(c_1v_s^2+c_2v_s\right)ds+c_3v_T^2}\middle|\mathcal{F}_t\right] = A(t)\,e^{B(t)v_t^2+C(t)v_t},$$

where

$$\begin{aligned}A(t) &= \frac{1}{\sqrt{\Psi(\gamma_1,\gamma_2)}}\\ &\quad\times\exp\left[\frac{\kappa(T-t)}{2} + \frac{\kappa^2\theta^2\gamma_1^2-\gamma_3^2}{2\xi^2\gamma_1^3}\left[\frac{\sinh\{\gamma_1(T-t)\}}{\Psi(\gamma_1,\gamma_2)} - \gamma_1(T-t)\right]\right.\\ &\qquad\left.+\frac{(\kappa\theta\gamma_1-\gamma_2\gamma_3)\gamma_3}{\xi^2\gamma_1^3}\left[\frac{\cosh\{\gamma_1(T-t)\}-1}{\Psi(\gamma_1,\gamma_2)}\right]\right],\\ B(t) &= \frac{1}{2\xi^2}\left[\kappa - \frac{\gamma_1\Phi(\gamma_1,\gamma_2)}{\Psi(\gamma_1,\gamma_2)}\right]\end{aligned}$$

and

$$C(t) = \frac{1}{\xi^2\gamma_1}\left[\frac{\kappa\theta\gamma_1-\gamma_2\gamma_3+\gamma_3\Phi(\gamma_1,\gamma_2)}{\Psi(\gamma_1,\gamma_2)} - \kappa\theta\gamma_1\right]$$

with

$$\begin{aligned}\Phi(\gamma_1,\gamma_2) &= \sinh\{\gamma_1(T-t)\} + \gamma_2\cosh\{\gamma_1(T-t)\},\\ \Psi(\gamma_1,\gamma_2) &= \cosh\{\gamma_1(T-t)\} + \gamma_2\sinh\{\gamma_1(T-t)\}\end{aligned}$$

and

$$\gamma_1 = \sqrt{\kappa^2+2c_1\xi^2},\quad \gamma_2 = \frac{\kappa-2c_3\xi^2}{\gamma_1},\quad \gamma_3 = \kappa^2\theta - c_2\xi^2.$$

Proof. The proof appears in the appendix of [7]. □

Using the result obtained in Lemma 3.1 and 3.2, we can obtain the following theorem about a closed-form expression for the price of a European power-α quanto call option in currency Y with foreign strike price K whose payoff at maturity T is given by $\max\left(S_T^\alpha - K, 0\right)$. For convenience, we here put the predetermined fixed exchange rate to 1.

Theorem 3.3. *Let us denote the log-asset price by $x_t = \ln S_t^\alpha$. Under the assumptions of (3) and (4), the price of a European power-α quanto call option in currency Y with foreign strike price K and maturity T is given by*

$$C_q\left(t, S_t^\alpha\right) = \mathbb{E}_{\mathbb{Q}^Y}\left[S_T^\alpha \middle| \mathcal{F}_t\right] e^{-r^Y(T-t)} P_1 - K e^{-r^Y(T-t)} P_2,$$

where P_1, P_2 are defined by

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathbf{Re}\left[\frac{e^{-i\phi\ln K} f_j\left(\phi\right)}{i\phi}\right] d\phi$$

for $j = 1, 2$, in which

$$f_1\left(\phi\right) = \frac{e^{(1+i\phi)\left\{\alpha\left(r^X - q - \frac{\rho\xi}{2}\right)(T-t) + x_t - \frac{\rho\alpha}{2\xi} v_t^2\right\}}}{\mathbb{E}_{\mathbb{Q}^Y}\left[S_T^\alpha \middle| \mathcal{F}_t\right]} \mathbb{E}_{\mathbb{Q}^Y}\left[e^{-\int_t^T\left(m_1 v_s^2 + m_2 v_s\right)ds + m_3 v_T^2} \middle| \mathcal{F}_t\right]$$

with

$$m_1 = \alpha\left(1 + i\phi\right)\left\{\frac{1 - \alpha\left(1 - \rho^2\right)}{2} - \frac{\rho\kappa}{\xi}\right\}, \quad m_2 = \alpha\left(1 + i\phi\right)\left(\frac{\rho\kappa\hat{\theta}}{\xi} - \nu\sigma_{FX}\right),$$

$$m_3 = \frac{\rho\alpha}{2\xi}\left(1 + i\phi\right)$$

and

$$f_2\left(\phi\right) = \frac{e^{i\phi\left\{\alpha\left(r^X - q - \frac{\rho\xi}{2}\right)(T-t) + x_t - \frac{\rho\alpha}{2\xi} v_t^2\right\}}}{\mathbb{E}_{\mathbb{Q}^Y}\left[S_T^\alpha \middle| \mathcal{F}_t\right]} \mathbb{E}_{\mathbb{Q}^Y}\left[e^{-\int_t^T\left(n_1 v_s^2 + n_2 v_s\right)ds + n_3 v_T^2} \middle| \mathcal{F}_t\right]$$

with

$$n_1 = i\alpha\phi\left\{\frac{1 - \alpha\left(1 - \rho^2\right)}{2} - \frac{\rho\kappa}{\xi}\right\}, \quad n_2 = i\alpha\phi\left(\frac{\rho\kappa\hat{\theta}}{\xi} - \nu\sigma_{FX}\right), \quad n_3 = \frac{i\rho\alpha\phi}{2\xi}.$$

Proof. From the risk-neutral valuation, the price of a European power-α quanto call option in currency Y with foreign strike price K and maturity T is given by

$$C_q\left(t, S_t^\alpha\right) = e^{-r^Y(T-t)} \mathbb{E}_{\mathbb{Q}^Y}\left[\max\left(S_T^\alpha - K, 0\right) \middle| \mathcal{F}_t\right].$$

For a new risk-neutral probability measure $\tilde{\mathbb{Q}}^Y$, the Radon-Nikodým derivative of $\tilde{\mathbb{Q}}^Y$ with respect to $\mathbb{Q}^Y$ is defined by

$$\frac{d\tilde{\mathbb{Q}}^Y}{d\mathbb{Q}^Y} = \frac{S_T^\alpha}{\mathbb{E}_{\mathbb{Q}^Y}\left[S_T^\alpha \middle| \mathcal{F}_t\right]}$$

on $\mathcal{F}_T$. Thus, the price of a European power-α quanto call option can be rewritten as

$$\begin{aligned} C_q\left(t, S_t^\alpha\right) &= e^{-r^Y(T-t)} \mathbb{E}_{\mathbb{Q}^Y}\left[S_T^\alpha \mathbb{1}_{\left\{S_T^\alpha > K\right\}} - K \mathbb{1}_{\left\{S_T^\alpha > K\right\}} \middle| \mathcal{F}_t\right] \\ &= \mathbb{E}_{\mathbb{Q}^Y}\left[S_T^\alpha \middle| \mathcal{F}_t\right] e^{-r^Y(T-t)} \tilde{\mathbb{Q}}^Y\left(S_T^\alpha > K\right) - K e^{-r^Y(T-t)} \mathbb{Q}^Y\left(S_T^\alpha > K\right) \end{aligned}$$

$$= \mathbb{E}_{\mathbb{Q}^Y}\left[S_T^\alpha\middle|\mathcal{F}_t\right] e^{-r^Y(T-t)}P_1 - Ke^{-r^Y(T-t)}P_2$$

with the risk-neutralized probabilities P_1 and P_2. Now, putting $x_t = \ln S_t^\alpha$, the corresponding characteristic functions f_1 and f_2 can be represented as

$$\begin{aligned} f_1(\phi) &= \mathbb{E}_{\tilde{\mathbb{Q}}^Y}\left[e^{i\phi x_T}\middle|\mathcal{F}_t\right] \\ &= \frac{1}{\mathbb{E}_{\mathbb{Q}^Y}\left[S_T^\alpha\middle|\mathcal{F}_t\right]}\mathbb{E}_{\mathbb{Q}^Y}\left[e^{(1+i\phi)x_T}\middle|\mathcal{F}_t\right], \\ f_2(\phi) &= \mathbb{E}_{\mathbb{Q}^Y}\left[e^{i\phi x_T}\middle|\mathcal{F}_t\right]. \end{aligned}$$

On the other hand, applying the Itô formula to (3), we have

$$dx_t = \alpha\left(r^X - q + \nu\sigma_{\mathrm{FX}}v_t - \frac{1}{2}v_t^2\right)dt + \rho\alpha v_t dW_t^{\mathbb{Q}^Y} + \alpha\sqrt{1-\rho^2}v_t dW_t.$$

From (7), we obtain

$$f_1(\phi) = \frac{e^{(1+i\phi)\left\{\alpha\left(r^X-q-\frac{\rho\xi}{2}\right)(T-t)+x_t-\frac{\rho\alpha}{2\xi}v_t^2\right\}}}{\mathbb{E}_{\mathbb{Q}^Y}\left[S_T^\alpha\middle|\mathcal{F}_t\right]}\mathbb{E}_{\mathbb{Q}^Y}\left[e^{-\int_t^T\left(m_1v_s^2+m_2v_s\right)ds+m_3v_T^2}\middle|\mathcal{F}_t\right]$$

with

$$m_1 = \alpha(1+i\phi)\left\{\frac{1-\alpha\left(1-\rho^2\right)}{2} - \frac{\rho\kappa}{\xi}\right\}, \quad m_2 = \alpha(1+i\phi)\left(\frac{\rho\kappa\hat{\theta}}{\xi} - \nu\sigma_{\mathrm{FX}}\right),$$
$$m_3 = \frac{\rho\alpha}{2\xi}(1+i\phi).$$

Similarly, we also obtain

$$f_2(\phi) = \frac{e^{i\phi\left\{\alpha\left(r^X-q-\frac{\rho\xi}{2}\right)(T-t)+x_t-\frac{\rho\alpha}{2\xi}v_t^2\right\}}}{\mathbb{E}_{\mathbb{Q}^Y}\left[S_T^\alpha\middle|\mathcal{F}_t\right]}\mathbb{E}_{\mathbb{Q}^Y}\left[e^{-\int_t^T\left(n_1v_s^2+n_2v_s\right)ds+n_3v_T^2}\middle|\mathcal{F}_t\right]$$

with

$$n_1 = i\alpha\phi\left\{\frac{1-\alpha\left(1-\rho^2\right)}{2} - \frac{\rho\kappa}{\xi}\right\}, \quad n_2 = i\alpha\phi\left(\frac{\rho\kappa\hat{\theta}}{\xi} - \nu\sigma_{\mathrm{FX}}\right), \quad n_3 = \frac{i\rho\alpha\phi}{2\xi}.$$

Here, each value of the above risk-neutral expectation was obtained in previous lemmas.

By having closed-form expressions for the characteristic functions f_1 and f_2, the Fourier inversion formula allows us to compute the probabilities P_1 and P_2 as follows:

$$P_j = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \mathbf{Re}\left[\frac{e^{-i\phi\ln K}f_j(\phi)}{i\phi}\right]d\phi$$

for $j = 1, 2$. □

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